Ordering Decimals

Put in ascending order: 14.61, 4.6, 14.06, 0.46, 4.61

Start by comparing the numbers in front of the decimal point – 0.46 is the smallest

Comparing 4.6 & 4.61 look after the point – 6 in both, but in the next column 1 is bigger than 0.

Comparing 14.61 & 14.06 – the first digits after the point are 0 and 6, so 6 is bigger.

Answer = 0.46, 4.6, 4.61, 14.06, 14.61

Multiplying Decimals

Use long multiplication to calculate the answer without the decimal point/s. Then use place value to correctly place the decimal point.

$$\begin{array}{r} 228 \\ \times \quad 7 \\ \hline 1596 \\ \hline {\scriptstyle 1\ 5} \end{array}$$

Answer

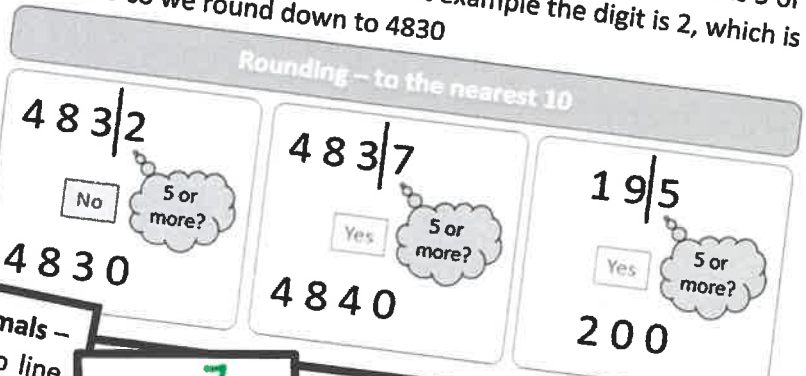
$2.28 \times 7 = 1596 \div 100 = \mathbf{15.96}$

Dividing Decimals

If dividing a decimal by an integer then use long division also known as the bus-stop method.

Rounding whole numbers

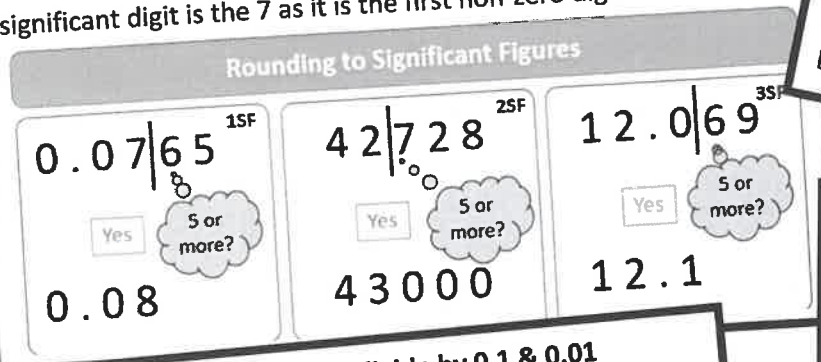
To round to the nearest 10 we draw a line between the tens and units column. We need to decide if the digit to the right of the line is 5 or more, if it is we round up. In the first example the digit is 2, which is less than 5 so we round down to 4830



Rounding to significant figures

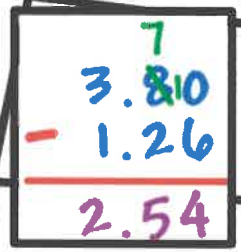
In the number 4691 the most significant digit is the 4 as it represents 4000. When rounding to 1sf we draw a vertical line after the 4, like this 4|691. As always with rounding we look at the next digit, in this case the 6 if it is 5 or more, we round up. 6 is 5 or more so we round the 4691 up to 5000. The answer only has 1sf

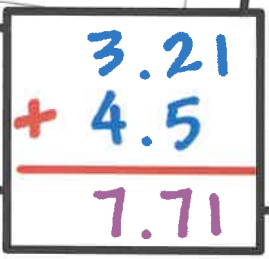
If the number is less than 1, for example 0.0765 then the most significant digit is the 7 as it is the first non-zero digit.



Adding or Subtracting Decimals –

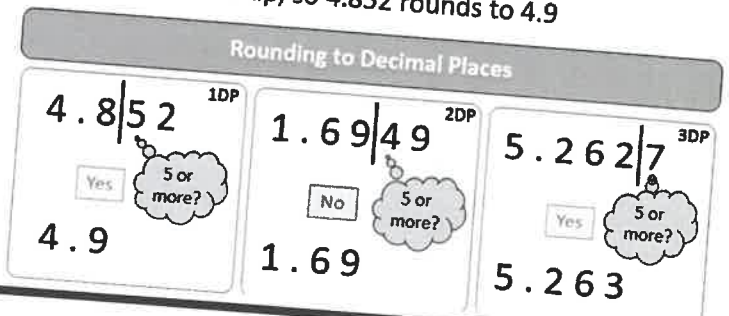
You must use place value to line up the numbers, ensure that the corresponding columns are in a line. Add place holders, i.e. 0's, until the numbers have the same length. Add or subtract the numbers using column addition. Look at the example to the right.





Rounding decimals

The first decimal place is the first digit after the decimal point. In the example below, 8 is in the first decimal place. We draw a line after this digit and then decide if the next digit is 5 or more. In this case we round up, so 4.852 rounds to 4.9



Multiply and divide by 0.1 & 0.01

Multiplying by 0.1 is the same as dividing by 10.

Multiplying by 0.01 is the same as dividing by 100.

Dividing by 0.1 is the same as multiplying by 10.

Dividing by 0.01 is the same as multiplying by 100.

Estimating

Estimating - To estimate or approximate we round each number to 1 significant figure first, and then perform the calculation.

$$\frac{312 \times 5.94}{2.03} \overset{\text{'approximately equal to'}}{\approx} \frac{300 \times 6}{2} = 900$$

$$\frac{36.84 \times 8.932}{3.301} \approx \frac{40 \times 9}{3} = 120$$

Percentage of an amount 10%, 1% and 50 %

To calculate 50% you divide by 2 because 50% = ½.
To calculate 25% you divide by 4 because 25% = ¼.
To calculate 10% you divide by 10 because 10% = $^1/_{10}$
To calculate 1% you divide by 100 because 1% = $^1/_{100}$

Examples:

10 % of 60 = 6
60 ÷ 10 = 6

50% of 24 = 12
24 ÷ 2 = 12

1% of 350 = 3.5
350 ÷ 100 = 3.5

25% of 40 = 10
40 ÷ 4 = 10

Adding and subtracting negative numbers

Adding moves us up the number line
Subtracting moves us down

$6 - 11 = -5$ because we start at 6 and move down 11 times

$-3 + 10 = 7$ because we start at – 3 and move up 10 units

$-1 + (-5) = -6$ because adding a negative is the same as subtracting

$-5 - 4 = -9$ because we start at -5 and move down 4 times

$-5 - (-4) = -5 + 4 = -1$ because subtracting a negative means add



Multiplier – is used to increase or decrease by a percentage.

To increase by 4%

100% + 4% = 104% which as a decimal is 1.04

Multiplying by 1.04 adds 4% to the original number

To decrease by 7%

100% - 7% = 93% which as a decimal is 0.93

Multiplying by 0.93 subtracts 7% off the original number

Increase by 80%, multiply by 1.8

Increase by 14%, multiply by 1.14

Decrease by 17%, multiply by 0.83

Percentage increase and decrease

To decrease 30 by 20%?

10% of 30 is 3

20% of 30 is 6

To decrease 30 by 20%

30 – 6 = 24

To increase 68 by 5%

10% of 68 = 6.8

5% of 68 = 6.8 ÷ 2 = 3.4

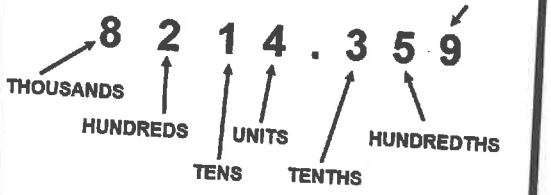
Because 5% is half of 10%

68 + 3.4 = 71.4

Or we could have used a multiplier

Place Value

Look at this number...
What does each digit mean?



BIDMAS

The order in which operations such as + − × ÷ are performed can be determined using BIDMAS

- Brackets
- Indices
- Division and Multiplication
- Addition and subtraction

Examples:

$4 + 2 \times 3 = 4 + 6 = 10$ because we multiply before we add. The other way round would give a different and incorrect answer

$6^2 - 3 \times (2 + 7) = 6^2 - 3 \times 9 = 36 - 3 \times 9 = 36 - 27 = 9$ because we calculate the brackets first, then work out the indices (powers or roots). This is followed by multiplication and finally subtracting!

Multiplying and dividing negative numbers

$-12 \times 3 = -36$ because one negative, when multiplied gives a negative answer

$-2 \times -8 = 16$ because two negatives, when multiplied give a positive answer

$28 \div -4 = -7$ because one negative, when multiplied gives a negative answer

$-32 \div -4 = 8$ because two negatives, when multiplied give a positive answer

A **TERM** is an element of an expression, equation or function such as 2ab, 5π or x^2y

An **EXPRESSION** is a combination of letters, numbers and symbols without an equals sign such as 2ab, $5\pi + 4$, or $2(x^2y + 7)$

An **EQUATION** is a combination of letters, numbers and symbols with an equals sign

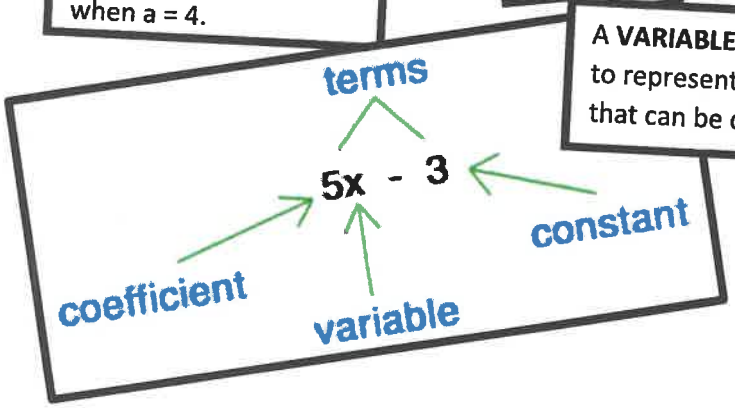
An **INDENTITY** is true for all values, for example $2a \equiv a + a$.

$3a = 12$ is NOT an identity as it is only true when $a = 4$.

A **CONSTANT** is a quantity or parameter that does not change its value.

A **COEFFICIENT** is a numerical quantity placed before a variable.

A **VARIABLE** is a letter to represent a value that can be changed



Expanding brackets

Multiply everything inside the bracket by the number and / or variable in front

$4(2a + 7) = 4 \times 2a + 4 \times 7 = 8a + 28$

$6a(a - 3b) = 6a \times a + 6a \times -3b = 6a^2 - 18ab$

Expand and simplify

First expand the brackets then simplify by collecting 'like' terms

$3(a + 3) + 2(a - 6) = 3a + 9 + 2a - 12$ when expanded, this simplifies to $5a - 3$

Expanding double brackets – higher sets only

Everything in the first bracket is multiplied by everything in the second bracket.

$(a + 3)(a + 5) = a(a + 5) + 3(a + 5)$

which expands to give: $a^2 + 5a + 3a + 15$

which simplifies to give: $a^2 + 8a + 15$

Collecting 'like' terms

'Like' terms contain the same variable raised to the same power, they can be collected together.

$5t - t$ can be shortened to $4t$

$t^2 + t^2 + t^2$ can be shortened to $3t^2$

$3a + 3b + 5a - b = 8a + 2b$

$a + a = 2a$

Substitution

Replacing a letter in an expression or formula with a value and calculating the result

What is the value of $4a + 2$ when $a = 5$

$(4 \times 5) + 2 = 20 + 2 = 22$

What is the value of $5b - 3$ when $b = -2$

$(5 \times -2) - 3 = -10 - 3 = -13$

Calculate $ab - c$ when $a = 4$, $b = 3$ and $c = -1$

$(4 \times 3) - (-1) = 12 + 1 = 13$

Formula Cooking time for Chicken is 30 minutes per kilogram (Kg), plus 40 minutes. What is the cooking time for a Chicken that weighs 2.5 Kg?

Cooking time = $30 \times$ weight + 40

$T = 30w + 40$

$T = (30 \times 2.5) + 40 = 75 + 40 = 115$ minutes.

Simplifying expressions

a x b can be written as ab

3 x t can be written as 3t

t x t can be written as t^2

t ÷ s can be written as $\frac{t}{s}$

$\frac{8a}{2}$ can be simplified to 4a

Factorising expressions – Writing an expression using brackets. Find the highest common factor of the terms in an expression, this goes in front of the brackets. To fill the brackets, find what is multiplied by the HCF to create the original expression.

$4a - 12 = 4(a - 3)$, 4 is the HCF of 4 & 12

$9b + 12 = 3(3b + 4)$, 3 is the HCF of 9 & 12

Solving one-step equations

Inverse operations perform the opposite procedure.
Addition and subtraction are inverse operations.
Multiplication and division are inverse operations.

$a + 4 = 5$

-4 -4

$a = 1$

$\frac{w}{3} = 15$

$\times 3$ $\times 3$

$w = 45$

$6t = 12$

$\div 6$ $\div 6$

$t = 2$

$b - 9 = 3$

$+9$ $+9$

$b = 12$

Solve with variables on both sides

Look at the coefficients first. Subtract the smallest coefficient from both sides.

$3a + 1 = 5a - 4$

$-3a$ $-3a$

$1 = 2a - 4$

$+4$ $+4$

$5 = 2a$

$\div 2$ $\div 2$

$2.5 = a$

Multiplying and dividing expressions

$5 \times 2p = 10p$ $\quad c \times e = ce$

$2a \times 3b = 6ab$ $\quad 5a \times 4a = 20a^2$

Always write the letters in alphabetical order

$\frac{5c}{20} = \frac{c}{4}$ we can divide the numerator and denominator by the HCF, in this case 5.

$\frac{b^2}{4b} = \frac{b}{4}$ [dividing by b] $\quad \frac{5ab}{10b} = \frac{a}{2}$ [dividing by 5 & b]

SOLVE - Work out the number represented by the variable.

FORMULA - an equation that is used to calculate a specific value such as area, volume, speed, pressure, etc. It shows the relationship between different variables.

Writing expressions

5 less than n can be written as $n - 5$

3 more than n is $n + 3$ or $3 + n$

4 multiplied by n is $4n$ the number must come before the variable

n divided by 2 is $\frac{n}{2}$

8 divided by n is $\frac{8}{n}$

n multiplied by n is written as n^2, this is because the n is being squared

n multiplied by 3 and then subtract 2 is $3n - 2$

n add 5, divided by 4 is $\frac{n+5}{4}$, this is because the 5 is added before dividing by 5

Solving two-step equations

$3a - 4 = 2$

$+4$ $+4$

$3a = 6$

$\div 3$ $\div 3$

$a = 2$

$25 = 4a + 1$

-1 -1

$24 = 4a$

$\div 4$ $\div 4$

$6 = a$

Solving equations with brackets

$4(t - 5) = 4$

Expand the bracket

$4t - 20 = 4$

$+20$ $+20$

$4t = 24$

$\div 4$ $\div 4$

$t = 6$

$2(t + 3) = 4(t - 2)$

Expand both brackets

$2t + 6 = 4t - 8$

$-2t$ $-2t$

$6 = 2t - 8$

$+8$ $+8$

$14 = 2t$

$7 = t$

Average – a single number taken as representative of a list of numbers

Mean – add the numbers together, then divide by how many numbers there were.
Mode – the number occurs the most
Median – put the number in order and find the middle.

Range – a measure of spread. What is the biggest value subtract the smallest

Continuous data will have infinite number of possible values within the selected range such as height or time.

Discrete data refers to data that has only a select number of values, such as shoe sizes. You can only be a size 6, 6.5, 7, 7.5 etc. You can't be a size 6.1

Averages

8 7 5 2 5 9

Mean = $(8 + 7 + 5 + 2 + 5 + 9) \div 6 = 6$

Mode = 5, it is the most common number

Median – written in order 2, 5, 5, 7, 8, 9
The middle number is 6, it is in between 5 and 7

Range

8 7 5 2 5 9

Range = biggest – smallest = 9 – 2 = 7

Averages of grouped data

Class Interval	*Mid-point*	*Frequency*	*Mid-point × Frequency*
$140 \le h < 150$	145	6	145 × 6 = 870
$150 \le h < 160$	155	16	155 × 16 = 2480
$160 \le h < 170$	165	21	165 × 21 = 3465
$170 \le h < 180$	175	8	175 × 8 = 1400
	Totals	51	8215

The modal class is $160 \le h < 170$ because it has the largest frequency, 21 therefore it is the most common.

To estimate the mean add two columns, one called midpoint and the other called fx which represents the mid-point × frequency columns

The mean can be estimated by dividing the total of the fx column by the total frequency. Using the above example 8215 ÷ 51 = 161.07

Frequency table

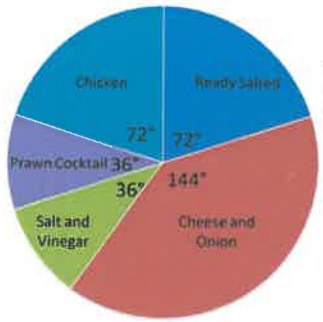
The table below shows the frequency (number of people) who chose each colour as their favourite.

Colour	Tally	Frequency
Red	𝍸	10
Blue	𝍸 |	8
Yellow	𝍸 |||	9
Green	𝍸 𝍸 |||	13

Pie Charts

A pie chart displays data using sectors of a circle. The bigger the sector the larger the frequency. The circle has 360°. So 360° represents the whole population. If we use the frequency table above:
There are 40 people, so each person is $\frac{360}{40} = 9°$
The 8 people who chose blue = 8 × 9° = 72°
So the sector for blue will have an angle of 72°.

Here is a completed pie chart about people's favourite crisp flavour:



Statistical investigations

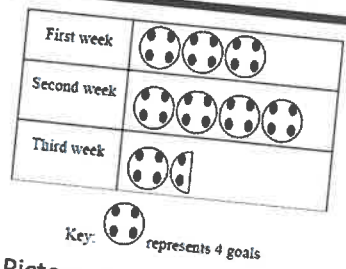
To complete a statistical investigation:

Specify a problem – what is 7e4's favourite colour?

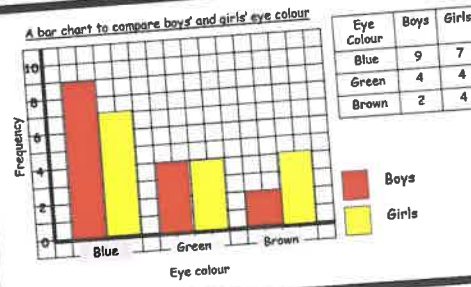
Collect data – you can use a frequency table like the one to the left.

Display the data – use a bar chart, pictogram or pie chart like the examples shown below

Write a conclusion to your investigation – which colour was the most popular



Pictograms represents data using a picture. Above, 1 ball represents 4 goals



Eye Colour	Boys	Girls
Blue	9	7
Green	4	4
Brown	2	4

Denominator - the number of equal parts that the whole has been split into. In $^3/_4$ the denominator, 4 tells us the whole has been split into 4 parts or quarters.

Numerator - In $^3/_4$ the numerator, 3 tells us how many quarters we have.

Equivalent - Two fractions are equivalent if they are equal

Common denominator – fractions can only be compared, added or subtracted if they are written with the same denominator

Mixed number – a number consists of an integer (whole number) and a fraction, for example $3\frac{2}{5}$.

Adding and subtract fractions

You must use common denominators!!

$$\frac{3}{4} + \frac{1}{3} = \frac{3 \times 3}{4 \times 3} + \frac{1 \times 4}{3 \times 4}$$

$$= \frac{9}{12} + \frac{4}{12}$$

$$= \frac{13}{12} = 1\frac{1}{12}$$

$$\frac{3}{4} - \frac{1}{3} = \frac{3 \times 3}{4 \times 3} - \frac{1 \times 4}{3 \times 4}$$

$$= \frac{9}{12} - \frac{4}{12}$$

$$= \frac{5}{12}$$

Multiplying and dividing fractions

$\frac{2}{5} \times \frac{6}{7} = \frac{2 \times 6}{5 \times 7} = \frac{12}{35}$

$\frac{1}{4} \times \frac{2}{3} = \frac{1 \times 2}{4 \times 3} = \frac{2}{12} =$ reduces to $\frac{1}{6}$

$\frac{2}{5} \div \frac{2}{3} = \frac{2}{5} \times \frac{3}{2} = \frac{2 \times 3}{5 \times 2} = \frac{6}{10} = \frac{3}{5}$

take the reciprocal of the divisor

$\frac{4}{7} \div 2 = \frac{4}{7} \times \frac{1}{2} = \frac{4 \times 1}{7 \times 2} = \frac{4}{14} = \frac{2}{7}$

Simplify $\frac{15}{20}$

Find the highest common factor of the numerator and denominator

Then divide both by the HCF $\frac{15}{20}$ $\boxed{\div 5}$ $\frac{3}{4}$

Four operations with mixed numbers

① $2\frac{5}{6}$ multiply these... 2×6

② $2\frac{5}{6}$ ← then add this to the product $2 \times 6 + 5$

③ $2 \times 6 + 5 = 17$ this is your new numerator

So, $2\frac{5}{6} = \frac{17}{6}$

$3\frac{2}{5} - 1\frac{4}{7} = \frac{17}{5} - \frac{11}{7}$ changes to improper fractions

$= \frac{17 \times 7}{5 \times 7} - \frac{11 \times 5}{7 \times 5} = \frac{119}{35} - \frac{55}{35}$ write with a common denominator

$= \frac{119 - 55}{35} = \frac{64}{35}$

Change $\frac{13}{6}$ to a mixed number

$13 \div 6 = 2$ R 1

$\frac{13}{6} = 2\frac{1}{6}$

An improper fraction has a numerator that is larger than its denominator.

What is $\frac{3}{5}$ of £350?

Divide by the denominator

$350 \div 5 = 70$

Then multiply by the numerator

$70 \times 3 = 210$

So $\frac{3}{5}$ of £350 = £210

Write 35 as a fraction of 150

Put the first number as the numerator

and the second number as the denominator.

Then simplify if possible. $\frac{35}{150}$ $\boxed{\div 5}$ $\frac{7}{30}$

Converting decimals to fractions

Use the place value of the last digit

$0.3 = \frac{3}{10}$ $0.03 = \frac{3}{100}$ $0.13 = \frac{13}{100}$

If possible – simplify:

$0.15 = \frac{15}{100} = \frac{3}{20}$ $0.48 = \frac{48}{100} = \frac{12}{25}$

Some key conversions to remember:

$0.5 = \frac{1}{2}$ $0.25 = \frac{1}{4}$

$0.75 = \frac{3}{4}$ $0.\dot{3} = \frac{1}{3}$

$0.125 = \frac{1}{8}$

Convert fractions to percentages

A percentage is out of 100, therefore we need to write the fraction with a denominator of 100.

$\frac{7}{10} = \frac{70}{100} = 70\%$

$\frac{3}{5} = \frac{60}{100} = 60\%$

$\frac{9}{20} = \frac{45}{100} = 45\%$

$\frac{11}{25} = \frac{44}{100} = 44\%$

For denominators that are not a factor of 100 we need to use long division or a calculator if allowed.

$\frac{5}{8} = 5 \div 8$

$= 0.625 = 62.5\%$

Convert percentages to fractions

A percentage is out of 100, therefore it can be written as a fraction with a denominator of 100; always simplify.

$30\% = \frac{30}{100} = \frac{3}{10}$ $55\% = \frac{55}{100} = \frac{11}{20}$

$32\% = \frac{32}{100} = \frac{8}{25}$ $2\% = \frac{2}{100} = \frac{1}{50}$

Convert percentages to decimals

To convert percentages to decimals divide by 100

$30\% = 30 \div 100 = 0.3$

$65\% = 65 \div 100 = 0.65$

$9\% = 9 \div 100 = 0.09$

$34.5\% = 34.5 \div 100 = 0.345$

Simplify – divide the numerator and denominator by their HCF

Converting - turns a number a number into a different form, for example a fraction can be converted into a decimal or percentage.

Convert decimals to percentages

You can multiply by 100 to turn a decimal or fraction into a percentage

$0.3 = 30\%$ $0.03 = 3\%$

$0.13 = 13\%$

Some key conversions to remember:

$0.5 = 50\%$ $0.25 = 25\%$

$0.75 = 75\%$ $0.\dot{3} = 33.\dot{3}\%$

Convert fractions to decimals

To convert a fraction to a decimal write the fraction with a denominator of 10, 100 or 1000.

$\frac{1}{5} = \frac{2}{10} = 0.2$ $\frac{3}{20} = \frac{15}{100} = 0.15$

$\frac{7}{25} = \frac{28}{100} = 0.28$ $\frac{7}{8} = \frac{875}{1000} = 0.875$

If the fraction cannot be written in this way, then divide the numerator by the denominator (long division or a calculator)

$\frac{3}{8} = 3 \div 8 = 0.375$ $\frac{2}{3} = 2 \div 3 = 0.\dot{6}$

Ordering fractions

Write them with the same denominator

$\frac{3}{5}\ \frac{7}{10}\ \frac{1}{2}\ \frac{13}{20}$ can be written as $\frac{12}{20}\ \frac{14}{20}\ \frac{10}{20}\ \frac{13}{20}$

So when written in order: $\frac{1}{2}\ \frac{3}{5}\ \frac{13}{20}\ \frac{7}{10}$

Ordering fractions, decimals and percentages

Convert everything to a percentage

$\frac{3}{5}$ 58% 0.59 can be written as 60% 58% 59%

So when written in order: 58% 0.59 $\frac{3}{5}$

Simplify Ratios

A ratio can be simplified by dividing each number by their highest common factor.

15 : 35 can be divided by 5 to give 3 : 7

30 : 24 can be divided by 6 to give 5 : 4

Simplifying with different units

Simplify 5cm : 4m. Remember 1m = 100cm

First convert to the same units 5cm : 400 cm

Then the units can be removed 5 : 400

Now we can divide by 5 to simplify to 1 : 80

Recipes

To make 4 slices of pizza	
100 ml	tomato sauce
300 g	plain flour
160 g	mozzarella cheese
2	garlic cloves
1 handfuls	basil

To work out the ingredients needed for 8 slices we would double the quantities of each ingredient.

To work out the ingredients needed for 5 slices, we first need to work out how much one slice needs.

For tomato sauce

One slice = 100 ÷ 4 = 25ml

Five slices = 25 × 5 = 125ml

Sharing in a ratio

Share £10 between Steve and Jon in the ratio 3 : 2

Steve : Jon

There are 5 parts altogether. £10 ÷ 5 = £2.

So each part is worth £2.

Steve : Jon

2 2 2 | 2 2

Therefore Steve gets 3 × £2 = £6

Jon gets 2 × £2 = £4

Unitary Method

When prices are in proportion we can find the cost of buying a different quantity by first finding the price of one item.

If 4 pens cost £1.20, what do 7 pens cost?

4 pens = £1.20

1 pen = £1.20 ÷ 4 = 30p

7 pens = 7 × 30p = £2.10

Simplify – divide all components of the ratio by their HCF.

Converting currency -

if £1 is worth $1.24

When exchanging £30 you will receive

30 ×1.24 = $37.20

When exchanging $30 you will receive

30 ÷ 1.24 = £24.19

Best buy

OFFICE DEALS	PAPER WORLD
Packs of 20 folders £10.80	Pack of 15 folders £8.40

To decide which the best value is, we need to compare the same quantity.

If we bought 60 folders, it would cost:

3×£10.80 = £32.40 at Office Deals

4 ×£8.40 = £33.60 at Paper World.

Therefore Office Deals is cheaper.

Alternatively we could find how much 1 folder costs at each store.

At Office Deals £10.80 ÷ 20 = £0.54

At Paper World £8.40 ÷ 15 = £0.56

Again Office Deals is cheaper.

Using a ratio to find what fraction each person gets

In the ratio 3 : 5 there are 8 parts (3 + 5 = 8)

The person who receives 3 parts is getting 3 out of 8 or $^3/_8$ this means that the second person gets $^5/_8$

Solving problems with ratios

When solving ratio problems it is often easiest to draw boxes to represent parts. For example: Sabrina and Vicky share money in the ratio 2 : 5. Vicky gets an extra £15.



If the 3 parts are worth £15 then each part is £15 ÷ 3 = £5

Therefore filling in the boxes shows us that

Vicky gets 2 × £5 = £10 and Sabrina gets 5 × £5 = £25