Factorising using the ac rule

The general form of a quadratic is $ax^2 + bx + c$

So, for $3x^2 + 10x - 8$, $a = 3$, $b = 10$, $c = -8$

First find ac, $3 \times -8 = -24$. So $ac = -24$

We now want two numbers that multiply to give -24 and that add to give 10 (the value of b).

$-2 \times 12 = -24$ and $-2 + 12 = 10$

We re-write $3x^2 + 10x - 8$ using the $-2 + 12$ instead of the 10, so $3x^2 + 12x - 2x - 8$

Now lets factorise! $3x^2 + 12x = 3x(x + 4)$ and $-2x - 8 = -2(x + 4)$. Notice the brackets are the same for both.

$$3x^2 + 12x - 2x - 8 = 3x(x + 4) - 2(x + 4)$$

$$= (3x - 2)(x + 4)$$

Solving using the quadratic formula

Not all quadratics factorise! If it's on the calculator papers you can use this formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$3x^2 + 10x - 8$, $a = 3$, $b = 10$, $c = -8$

Putting the numbers into the formula gives:

$$x = \frac{-10 \pm \sqrt{10^2 - 4 \times 3 \times -8}}{2 \times 3}$$

This gives two answers (quadratics always do) one answer when you use the + and one answer when you use the - button.

You should get $x = {}^2/_3$ or $x = -4$, the same answers as when we factorised.

Completing the square

You may be asked to write a quadratic in the form $(x + p)^2 + q$

If $(x + 3)^2 = (x + 3)(x + 3) = x^2 + 6x + 9$

Then $x^2 + 6x + 10 = (x + 3)^2 + 1$
as 10 is 1 greater than 9

When we complete the square the value in the brackets is always half the coefficient of x because $(x + a)^2 = (x + a)(x + a) = x^2 + ax + ax + a^2$
Which simplifies to give $x^2 + 2ax + a^2$

Therefore $x^2 + 8x + 10 = (x + 4)^2 + q$
4 is half of 8, the coefficient of x

Knowing that $(x + 4)^2 = x^2 + 8x + 16$
and comparing this to $x^2 + 8x + 10$ ...
we have 6 more in the first expression, therefore:

$x^2 + 8x + 10 = (x + 4)^2 - 6$
as 16 is 6 more than 10

Solving by factorising

$3x^2 + 10x - 8 = 0$

If it doesn't equal 0 rearrange first

Factorising (see above) gives:

$(3x - 2)(x + 4) = 0$

So either $3x - 2 = 0$ or $x + 4 = 0$

Solving gives $x = {}^2/_3$ or $x = -4$

Solving by completing the square

Solve $x^2 + 8x + 10 = 0$

Complete the square $(x + 4)^2 - 6 = 0$

Add 6 to both sides $(x + 4)^2 = 6$

Square root both sides $x + 4 = \pm\sqrt{6}$
This gives two values, one positive and one negative. Think $\sqrt{9} = 3$ *or* -3

Subtract 4 from both sides $x = -4 \pm \sqrt{6}$
So $x = -4 + \sqrt{6}$ or $-4 - \sqrt{6}$

An alternative method to completing the square

$x^2 + 8x + 10 = (x + 4)^2 - 4^2 + 10 = (x + 4)^2 - 6$

The 4 is half of the 8 which is the coefficient of x

Another example:

$x^2 + 6x + 1 = (x + 3)^2 - 3^2 + 1 = (x + 3)^2 - 8$

And another:

$x^2 - 4x - 3 = (x - 2)^2 - (-2)^2 - 3 = (x - 2)^2 - 7$

Solve with variables on both sides

Look at the coefficients first. Subtract the smallest coefficient from both sides.

$3a + 1 = 5a - 4$

$-3a$ $-3a$

$1 = 2a - 4$

$+4$ $+4$

$5 = 2a$

$\div 2$ $\div 2$

$2.5 = a$

Solving equations with brackets

$4(t - 5) = 4$

Expand the bracket

$4t - 20 = 4$

$+20$ $+20$

$4t = 24$

$\div 4$ $\div 4$

$t = 6$

Expanding double brackets

Everything in the first bracket is multiplied by everything in the second bracket.

$(a + 3)(a + 5) = a(a + 5) + 3(a + 5)$

Which expands to give: $a^2 + 5a + 3a + 15$

Which simplifies to give: $a^2 + 8a + 15$

To expand triple brackets, higher tier only, expand the first two brackets the multiply this by the final bracket

$(a + 3)(a + 5)(a - 2) = (a^2 + 8a + 15)(a - 2)$

Expanding: $a^3 - 2a^2 + 8a^2 - 16a + 15a - 30$

Which simplifies to: $a^3 + 6a^2 - a - 30$

Factorise Quadratics

When we expanded $(a + 3)(a + 5)$ the answer was $a^2 + 8a + 15$. Notice $3 + 5 = 8$ and $3 \times 5 = 15$.

Therefore the sum of the numbers is the coefficient of 'a' and the product is the constant

To factorise $x^2 + 7x + 10$ we want two numbers that multiply to give 10 and add to give 7.

$2 \times 5 = 10$ and $2 + 5 = 7$ so the answer is $(x + 2)(x + 5)$

Rearranging – change the position of something, in this case a chosen variable. This is often called changing the subject. In the formula $y = 3x - 2$, y is the subject because it is on one side of the equals sign its own. We can rearrange to make x the subject.

Inequalities

The symbol $=$ is used when both sides are equal. The symbols $< > \leq \geq$ are used when the sides are unequal.

$x > 5$ means x is greater than 5

$x \leq 7$ means that x is less than or equal to 7

$1 \leq x < 4$ means that x is greater than or equal to 1 **AND** less than 4. If they ask for an integer that satisfies the inequality, we would say 1, 2 or 3

Make x the subject

$y = 3x - 2$

$+2$ $+2$

$y + 2 = 3x$

$\div 3$ $\div 3$

$\frac{y+2}{3} = x$

x is now the subject, because it is on its own on one side of the equals sign.

Rearranging when the variable appears twice

$$y = \frac{5x + 4}{x}$$

Multiply both sides by x

$xy = 5x + 4$

Put anything with an x in on one side

$xy - 5x = 4$

Factorise

$x(y - 5) = 4$

Divide by the contents of the brackets

$$x = \frac{4}{y - 5}$$

Solving quadratics

One way of solving a quadratic is factorising. Factorising writes a quadratic as two expressions multiplied together – like the example above.

Solve $x^2 + 4x - 12 = 0$

$6 \times (-2) = -12$ and $6 + (-2) = 4$

So, $(x + 6)(x - 2) = 0$

This means that $(x + 6) \times (x - 2) = 0$

Two numbers will multiply to give 0 if and only if one of the number is 0

So, either $x + 6 = 0$ or $x - 2 = 0$

So x can either be 2 or -6

Quadratics always have 2 solutions

Adding and subtract fractions

You must use common denominators!!

$$\frac{3}{4} + \frac{1}{3} = \frac{3 \times 3}{4 \times 3} + \frac{1 \times 4}{3 \times 4}$$

$$= \frac{9}{12} + \frac{4}{12}$$

$$= \frac{13}{12} = 1\frac{1}{12}$$

$$\frac{3}{4} - \frac{1}{3} = \frac{3 \times 3}{4 \times 3} - \frac{1 \times 4}{3 \times 4}$$

$$= \frac{9}{12} - \frac{4}{12}$$

$$= \frac{5}{12}$$

Change $\frac{13}{6}$ to a mixed number

$13 \div 6 = 2$ R 1

$\frac{13}{6} = 2\frac{1}{6}$

Converting decimals to fractions

Use the place value of the last digit

$0.3 = \frac{3}{10}$ $\quad 0.13 = \frac{13}{100}$

If possible – simplify:

$0.15 = \frac{15}{100} = \frac{3}{20}$

Convert between decimals and %s

You can multiply by 100 to turn a decimal or fraction into a percentage

$0.3 = 30\%$ $\quad 0.03 = 3\%$

$0.13 = 13\%$ $\quad 0.\dot{3} = 33.\dot{3}\%$

To convert a % to a decimal divide by 100

$30\% = 30 \div 100 = 0.3$

$65\% = 65 \div 100 = 0.65$

Four operations with mixed numbers

① $2\frac{5}{6}$ ← multiply these... 2×6

② $2\frac{5}{6}$ ← then add this to the product $2 \times 6 + 5$

③ $2 \times 6 + 5 = 17$ ← this is your new numerator

So, $2\frac{5}{6} = \frac{17}{6}$

$3\frac{2}{5} - 1\frac{4}{7} = \frac{17}{5} - \frac{11}{7}$ change to improper fractions

$= \frac{17 \times 7}{5 \times 7} - \frac{11 \times 5}{7 \times 5} = \frac{119}{35} - \frac{55}{35}$ write with a common denominator

$= \frac{119 - 55}{35} = \frac{64}{35}$

Convert fractions to percentages

- write the fraction with a denominator of 100.

$\frac{7}{10} = \frac{70}{100} = 70\%$ $\quad \frac{3}{5} = \frac{60}{100} = 60\%$

For denominators that are not a factor of 100 we need to use long division or a calculator if allowed.

$\frac{5}{8} = 5 \div 8 = 0.625 = 62.5\%$

Convert fractions to decimals

- write the fraction with a denominator of 10, 100 or 1000.

$\frac{3}{20} = \frac{15}{100} = 0.15$ $\quad \frac{7}{25} = \frac{28}{100} = 0.28$

If the fraction cannot be written in this way, then divide the numerator by the denominator (long division or a calculator)

$\frac{3}{8} = 3 \div 8 = 0.375$ $\quad \frac{2}{3} = 2 \div 3 = 0.\dot{6}$

Convert percentages to fractions

A percentage is out of 100, therefore it can be written as a fraction with a denominator of 100; always simplify.

$30\% = \frac{30}{100} = \frac{3}{10}$ $\quad 55\% = \frac{55}{100} = \frac{11}{20}$

$32\% = \frac{32}{100} = \frac{8}{25}$ $\quad 2\% = \frac{2}{100} = \frac{1}{50}$

Multiplying and dividing fractions

$\frac{2}{5} \times \frac{6}{7} = \frac{2 \times 6}{5 \times 7} = \frac{12}{35}$

$\frac{1}{4} \times \frac{2}{3} = \frac{1 \times 2}{4 \times 3} = \frac{2}{12} =$ reduces to $\frac{1}{6}$

$\frac{2}{5} \div \frac{2}{3} = \frac{2}{5} \times \frac{3}{2} = \frac{2 \times 3}{5 \times 2} = \frac{6}{10} = \frac{3}{5}$

take the reciprocal of the divisor

$\frac{4}{7} \div 2 = \frac{4}{7} \times \frac{1}{2} = \frac{4 \times 1}{7 \times 2} = \frac{4}{14} = \frac{2}{7}$

Algebraic Fractions

Simplify by factorising and/or cancelling terms. The $(x+3)$ is a common factor so we can divide the numerator and denominator by it.

$$\frac{2x+6}{x^2+5x+6}=\frac{2(x+3)}{(x+2)(x+3)}=\frac{2}{(x+2)}$$

Add or subtract by writing with a common denominator, like with normal fractions. $(x+5)(x+2)$ is the same as $(x+2)(x+5)$. Notice we don't need to expand the denominator.

$$\frac{2}{x+2}+\frac{3}{x+5}=\frac{2(x+5)}{(x+2)(x+5)}+\frac{3(x+2)}{(x+5)(x+2)}$$

$$=\frac{2x+10}{(x+2)(x+5)}+\frac{3x+6}{(x+5)(x+2)}=\frac{5x+16}{(x+2)(+5)}$$

Multiply and divide like with normal fractions, remember to simplify wherever possible.

$$\frac{x}{x+4}\times\frac{3x+12}{2x}=\frac{x(3x+12)}{2x(x+4)}$$

$$=\frac{3x+12}{2(x+4)}=\frac{3(x+4)}{2(x+4)}=\frac{3}{2}$$

There was a common factor of an x that could be cancelled. Then after factorising there was a common factor of $x+4$ which again could be cancelled.

When dividing, flip the second fraction, then multiply

Recurring decimals

Recurring decimals go on forever! They also repeat themselves, so we can write them using dots above the recurring (repeating) digits.

$$0.333333\ldots\ldots=0.\dot{3}$$

$$0.8333333\ldots\ldots=0.8\dot{3}$$

$$0.73737373\ldots\ldots=0.\dot{7}\dot{3}$$

All terminating and recurring decimals can be written as fractions, e.g. $0.\dot{3}=\frac{1}{3}$

Irrational Numbers

Irrational numbers are decimals that do not terminate (0.4, 0.35) or recur $0.\dot{3}$.

This means they cannot be written as a fraction.

The most famous irrational number is π

$\pi = 3.14159\ldots$ it goes on forever and never repeats

Convert recurring decimals to fractions

$$0.\dot{3}=\frac{3}{9}=\frac{1}{3} \qquad 0.\dot{7}\dot{3}=\frac{73}{99}$$

But it is likely that we will be asked something more challenging. For this we can use algebra.

$$0.8\dot{3}=0.8333\ldots\ldots$$

Let's call it x, $x=0.8333\ldots\ldots$

So, $10x=8.333\ldots\ldots$ and $100x=83.333\ldots$

$$\begin{array}{r} 100x = 83.333\ldots \\ -\ 10x = 8.333\ldots \\ 90x = 75 \end{array}$$

Try to line up the decimal points. If correct then all the digits after the points should match, this way the subtraction leaves an integer (whole number).

Solving the equation gives $x=\frac{75}{90}=\frac{15}{18}=\frac{5}{6}$

Prove that a recurring decimal equals a given fraction

An example of an exam question:

Prove that $0.7\dot{5}\dot{3}=\frac{373}{495}$

Let's call it x, $x=0.753753753\ldots\ldots$

So, $10x=7.535353\ldots$ $100x=75.35353\ldots$

and $1000x=753.5353\ldots$

$$\begin{array}{r} 1000x = 753.535353\ldots \\ -\ 10x = 7.535353\ldots \\ 990x = 746 \end{array}$$

Try to line up the decimal points. If correct then all the digits after the points should match, this way the subtraction leaves an integer (whole number).

Solving the equation gives $x=\frac{746}{990}=\frac{373}{495}$

Multiplier – is used to increase or decrease by a percentage.

To increase by 4%, 100% + 4% = 104% which as a decimal is 1.04
Multiplying by 1.04 adds 4% to the original number

To decrease by 7%, 100% - 7% = 93% which as a decimal is 0.93
Multiplying by 0.93 subtracts 7% off the original number

Increase by 80%, multiply by 1.8 Increase by 14%, multiply by 1.14

Decrease by 17%, multiply by 0.83

Simple interest – find the percentage and keep adding it on. £60 is invested for 4 years at 5% simple interest. 5% of £60 is £3. So if the interest was applied every year, for 4 years then the £60 would become 60 + 3 + 3 + 3 + 3 = £72

Compound interest – what banks use when you save money with them or take out a loan from them.

$$New\ Amount = Original\ amount \times multiplier^{years}$$

If I invest £2000 at a bank that gave 4% the after 3 years I would have: $2000 \times 1.04^3 = £2249.73$ always write your answer with 2 decimal places (pence).

Calculate percentage profit

If we know the original price and the new price, we can find the percentage that it has been changed by.

This might be called the percentage profit, percentage increase, percentage decrease, or percentage change – it's the same formula

$$Percentage\ profit = \frac{profit}{original\ cost} \times 100$$

$$Percentage\ increase = \frac{increase}{original\ price} \times 100$$

$$Percentage\ decrease = \frac{decrease}{original\ price} \times 100$$

$$Percentage\ change = \frac{change}{original\ price} \times 100$$

Find the amount the cost has changed by, divide it by the original value, then times by 100

Reverse percentages

If we are told the new price and the percentage it changed by, then we can find out the original price.

After a 40% increase my monthly phone bill is £42. Find the cost of my phone bill before the increase.

Method 1: use a multiplier

A 40% increase means the amount was multiplied by 1.4 to get the new cost, £42.

$$£x \xrightarrow{\times 1.4} £42$$

$$£42 \xrightarrow{\div 1.4} £x$$

To reverse the process we can divide by 1.3

$$£42 \div 1.4 = £30$$

Reverse percentages

If you are doing a non-calculator paper, then using a multiplier may be difficult

After a 30% increase my monthly phone bill is £42. Find the cost of my phone bill before the increase.

Method 2: use proportion

A 40% increase means the £42 is worth 100% + 40% = 140%

So, £42 = 140%

then 10% = £42 ÷ 14 = £3

therefore 100% = 10 × £3 = £30

Which is the same answer as method 1

Ordering fractions

Write them with the same denominator

$\frac{3}{5}\ \frac{7}{10}\ \frac{1}{2}\ \frac{13}{20}$ can be written as $\frac{12}{20}\ \frac{14}{20}\ \frac{10}{20}\ \frac{13}{20}$

So when written in order: $\frac{1}{2}\ \frac{3}{5}\ \frac{13}{20}\ \frac{7}{10}$

Ordering fractions, decimals and percentages

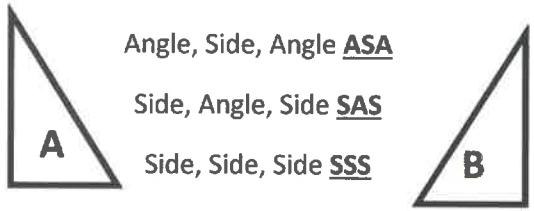
Convert everything to a percentage

$\frac{3}{5}$ 58% 0.59 can be written as 60% 58% 59%

So when written in order: 58% 0.59 $\frac{3}{5}$

Congruent = when two objects are the same size and shape, such as A and B. The shapes may have been rotated or reflected.

5 Rules to Prove 2 Triangles are Congruent:



Angle, Side, Angle **ASA**

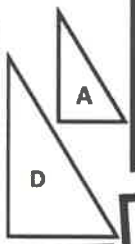
Side, Angle, Side **SAS**

Side, Side, Side **SSS**

Angle, Side, Angle **ASA**

Right-angle, Hypotenuse, Side **RHS**

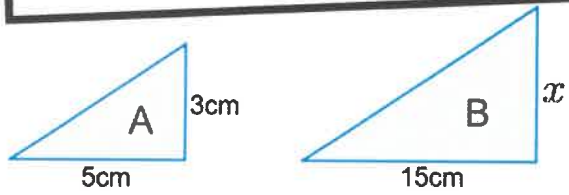
Similar = when two shapes are the same but are different sizes. They have the same angles but different lengths, such as A and D. D is an enlargement of A



Scale Factor – how many times bigger is one shape compared to another? To work out the scale divide one side by the corresponding side on the other shape.

Triangle B is $(15cm \div 5cm) = 3$ times larger than Triangle A.

Therefore $x = 3cm \times 3 = 9cm$

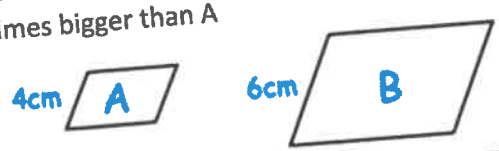


Area Scale Factor = (Length Scale Factor)^2

Volume Scale Factor = (Length Scale Factor)^3

Example: Parallelogram B is $6 \div 4 = 1.5$ times larger than Parallelogram A

That means the area of B is $1.5^2 = 2.25$ times bigger than A

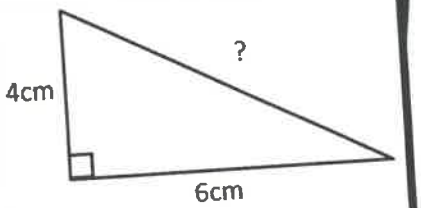


Pythagoras' Theorem

Pythagoras discovered that the square of the longest side is always equal to the sum of the squares of the two shorter sides $a^2 + b^2 = c^2$

The longest side c is called the hypotenuse

Finding the hypotenuse



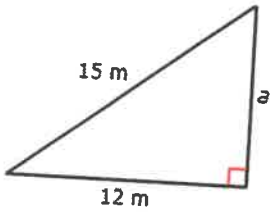
$4^2 + 6^2 = ?^2$

$52 = ?^2$

$\sqrt{52} = ?$

So the hypotenuse is 7.2cm (1dp)

Finding a shorter side



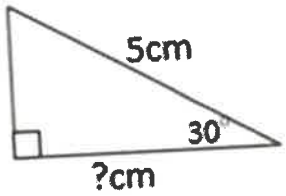
$a^2 + 12^2 = 15^2$

$a^2 + 144 = 225$

$a^2 = 81$ so, $a = \sqrt{81}$

So the height is 9m

Calculating a length



? is ADJ and 5cm is HYP so we are using the Cos formula

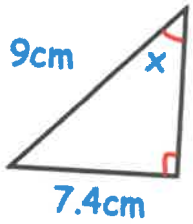
$$\cos(30) = \frac{?}{5}$$

Multiply both sides by 5

$5 \times \cos(30) = ?$

? = 4.33cm (2dp)

Finding an angle



7.4cm is OPP and 9cm is HYP so we are using the Sin formula

$$\sin(x) = \frac{7.4}{9}$$

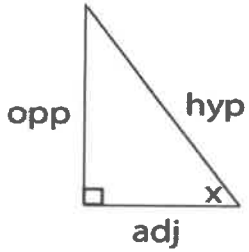
the inverse of sin is $\sin^{-1}$

$$x = \sin^{-1}\left(\frac{7.4}{9}\right)$$

$x = 55.3°$ (1dp)

Trigonometry (SOH CAH TOA)

Using the position of the angle (x) we can describe the sides as being opposite x, adjacent (next to) x and the hypotenuse which is the longest side that is always opposite the right angle. This gives 3 formulas, that you need to learn!



$$\sin(x) = \frac{opp}{hyp}$$

$$\cos(x) = \frac{adj}{hyp}$$

$$\tan(x) = \frac{opp}{adj}$$

Exact values to learn

	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Sine Rule

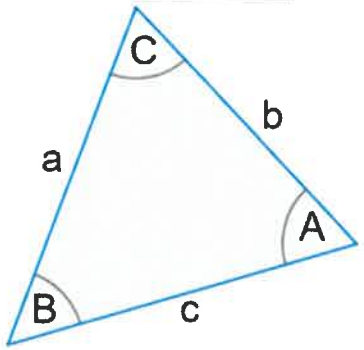
To find a length use:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

To find an angle use:

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

Labelling triangles



Cosine Rule

To find a length use:

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

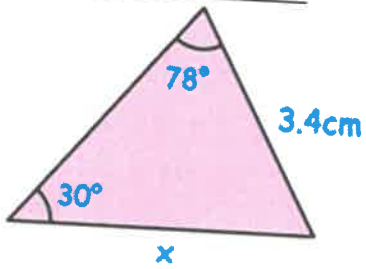
To find an angle use:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Area of a triangle

$$= \frac{1}{2}ab\sin C$$

Using the sine rule:



We have a an angle and its corresponding side, 30° and 3.4cm therefore use the sine rule

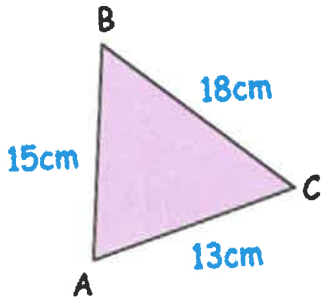
$$\frac{x}{\sin(78)} = \frac{3.4}{\sin(30)}$$

rearranging to get:

$$x = \frac{3.4}{sin(30)} \times sin(78)$$

$$x = 6.65\text{cm (3 sf)}$$

Using the cosine rule – if we have all 3 sides then we can use the cosine rule to find the angles. Finding A below...



$$\cos A = \frac{13^2 + 15^2 - 18^2}{2 \times 13 \times 15}$$

$$\cos A = \frac{70}{390}$$

$$A = \cos^{-1}\left(\frac{70}{390}\right) = 79.7°$$

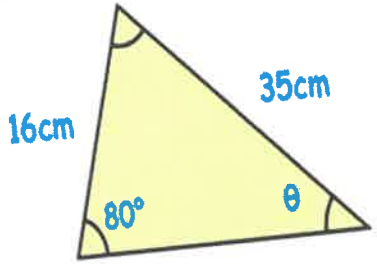
Using the sine rule: We have a an angle and its corresponding side, 80° and 35cm therefore use the sine rule

$$\frac{\sin\theta}{16} = \frac{\sin 80}{35}$$

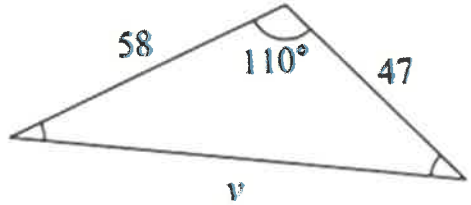
$$\sin\theta = \frac{\sin 80}{35} \times 16$$

$$\theta = \sin^{-1}\left(\frac{\sin 80}{35} \times 16\right)$$

$$\theta = 26.75°$$



Using the cosine rule



Here we have an angle in between two sides, we can use the cosine rule to find the third side, in this case v.

$$v^2 = 58^2 + 47^2 - 2 \times 58 \times 47 \times \cos(110)$$

$$v^2 = 7437.693 \ldots \ldots$$

$$v = \sqrt{7437.693 \ldots}$$

$$v = 86.24 \text{ (2 dp)}$$

Area of the triangle above ...

$$= \frac{1}{2} \times 58 \times 47 \times \sin 110$$

$$= 1280.8$$

Comparing 2 sets of data

For full marks we need to compare an average (mean or median) and a measure of spread (range or IQR). Find both first.

Male shoes sizes: 7, 8, 8, 10, 12
Median is 8. Range is 12 – 7 = 5.

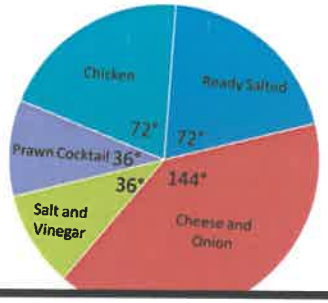
Female shoe sizes: 3, 4, 6, 7, 9
Median is 6. Range is 9 – 3 = 6.

Answer... 'On average the boys have bigger shoes (8 > 6). The girls feet are more spread out (6 > 5)

Pie Charts

A pie chart displays data using sectors of a circle. The bigger the sector the larger the frequency. The circle has 360°. So 360° represents the whole population.

If there are 40 people, each person is $\frac{360}{40} = 9°$
So if 8 people like chicken flavour = $8 \times 9° = 72°$
So the sector for chicken will be an angle of 72°.
Here is a completed pie chart about people's favourite crisp flavour:



Stem-and-leaf diagrams

6	2 3 3 5 7 9
7	1 4 5 8 8
8	1 1 2 4 6

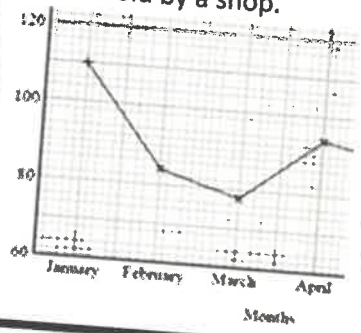
key: 6|2=62

Numbers can be organised like above, always use a key.

Range = 86 – 62 = 24
Mode = 78 (most common)
Median = 74.5 (middle values are 74 and 75)

Time-series graph

Shows a trend over time, below is number of shoes sold by a shop.



Averages of grouped data

Class Interval	Mid-point	Frequency	Mid-point × Frequency
$140 \leq h < 150$	145	6	145 × 6 = 870
$150 \leq h < 160$	155	16	155 × 16 = 2480
$160 \leq h < 170$	165	21	165 × 21 = 3465
$170 \leq h < 180$	175	8	175 × 8 = 1400
	Totals	51	8215

The modal class is $160 \leq h < 170$ because it has the largest frequency, 21 therefore it is the most common.

To estimate the mean add two columns, one called midpoint and the other called fx which represents the mid-point × frequency columns

The mean can be estimated by dividing the total of the fx column by the total frequency. Using the above example 8215 ÷ 51 = 161.07

Median from grouped data

The median is the middle value. To find which value is the middle, we +1 ÷ 2. On the example to the left 51 peoples heights were recorded. The median person is $(51 + 1) \div 2 = 26^{th}$ person. Whichever group contains the 26^{th} person is the median class.

The first group has 6 people.
The first 2 groups contain 6 + 16 = 22 people
The first 3 groups contain 22 + 21 = 43 people

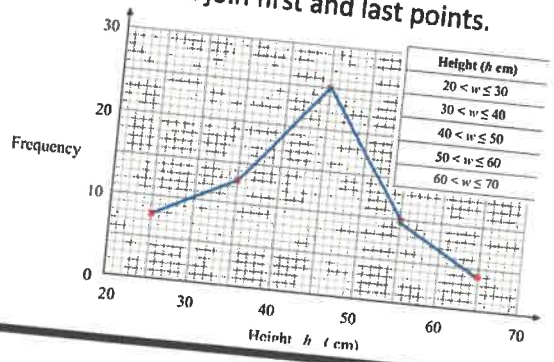
We have passed the 26^{th} person, so they must be in the third group. Answer ... $160 \leq h < 170$. Always write it how it appears in the table.

Use Back-to-back Stem-and-lead to compare two sets of data

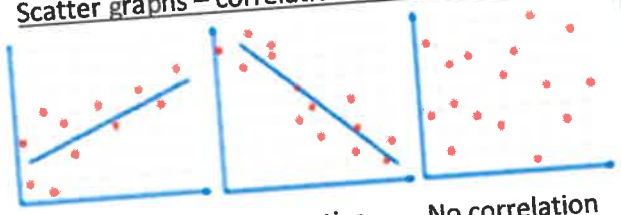
BOYS							GIRLS					
			8	5	0	3	5	9				
		7	4	0	1	0	5	7	9	9		
7	5	4	2	2	0	2	0	0	0	2	3	4
9	6	3	2	0	3	0	5					

Key:	4	1	5 means £14 for the boys and £15 for the girls

Frequency Polygons – plot at the midpoint, join with straight lines, don't join first and last points.



Scatter graphs – correlation and lines of best fit

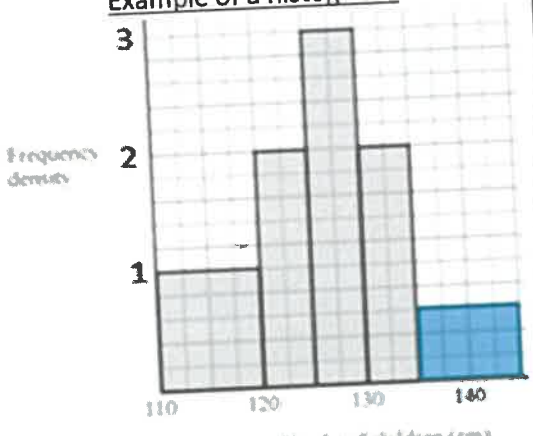


Positive Negative No correlation

There are 3 types of correlation (see above), they describe the relationship between the 2 variables. For example the first graph could be hours spent revising and test score. The more hours spent revising, the higher the test score.

If there is a correlation, then a line of best fit can be drawn. It must go in the direction of the trend with a similar number of points above and below.

Example of a histograms



Histograms

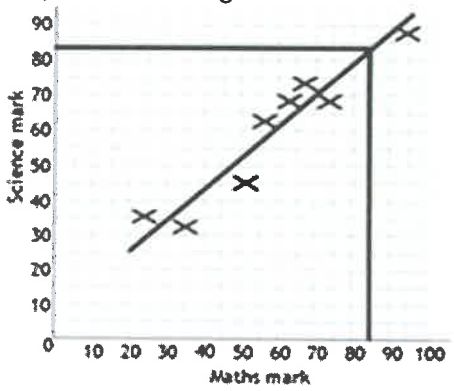
The area of each bar represents the frequency. The height of each bar is called the frequency density, which is found using: $Frequency\ density = \frac{Frequency}{Class\ width}$

On the example to the left, the final bar $(135 \leq h < 145)$ has a class width of 10 $(145 - 135)$ and a freq density of 0.6, therefore the bar represents $0.6 \times 10 = 6$ people

You may be asked to draw a histogram (you must find and plot the freq densities) or interpret a histogram (use the area of each bar to find the frequencies).

Using a line of best fit

If a student scored 85 on the Maths test, we can use the line of best fit to predict their Science mark. Looking at the example below, we would predict 83/84 marks using the line of best fit



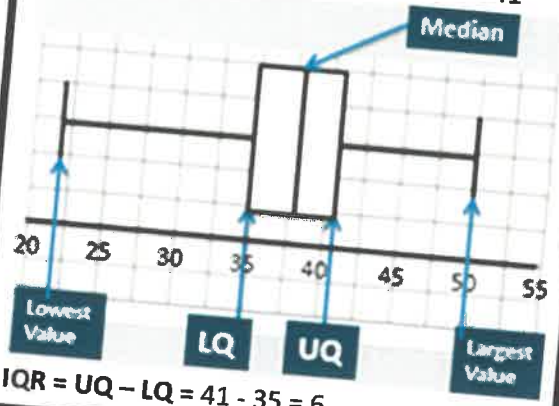
Quartiles and Box Plots

Ages of 19 people: 22, 30, 34, 35, 35, 35, 36, 37, 37, 38, 39, 39, 40, 40, 41, 42, 42, 48, 50

Median = (19+1)/2 = 10th value = 38

Lower Quartile = (19+1)/4 = 5th value = 35

Upper Quartile = LQ x 3 = 15th value = 41



IQR = UQ – LQ = 41 - 35 = 6

Cumulative Frequency

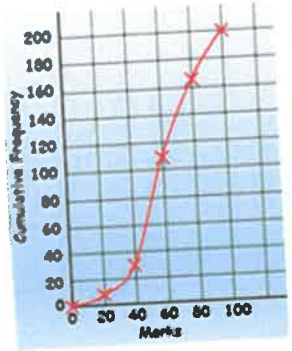
To find the cf values add the frequencies together.

$7 + 24 = 31,\ \ 7 + 24 + 83 = 114$ and so on

Mark	0 - 19	20 - 39	40 - 59	60 - 79	80 - 100
Frequency	7	24	83	52	34
Cumulative Frequency	7	31	114	166	200

We plot the cumulative frequencies at the highest value in the range. For example plot 31 at 39 marks.

The plot can then be used to make predictions (how many students scored less than 50?) or find quartiles.



Simplifying with different units

Simplify 5cm : 4m. Remember 1m = 100cm

First convert to the same units 5cm : 400 cm

Then the units can be removed 5 : 400

Now we can divide by 5 to simplify to 1 : 80

Using a ratio to find what fraction each person gets

In the ratio 3 : 5 there are 8 parts (3 + 5 = 8)

The person who receives 3 parts is getting 3 out of 8 or $^3/_8$ this means that the second person gets $^5/_8$

Writing equations for direct or indirect proportion

$\propto$ is the symbol of proportionality. k is the constant of proportionality. $y \propto x$ means y is proportional to x. This can be made into an equation using k to get $y = kx$.

y can be proportional to x, x^2, x^3 or $\sqrt{x}$
to get $y = kx$, $y = kx^2$ $y = kx^3$, $y = k\sqrt{x}$

y can be inversely proportional to x, x^2, x^3 or $\sqrt{x}$,
to get $y = \frac{k}{x}$, $y = \frac{k}{x^2}$, $y = \frac{k}{x^3}$, $y = \frac{k}{\sqrt{x}}$

Example: y is proportional to the square of x, when y is 72, x is 3. Find y when x is 4.

Using the first bit, $y = kx^2$
Substitute in the given values, $72 = k \times 3^2$
So, $k = 8$. Which gives the equation $y = 8x^2$

This equation can be used to find y given x
We are told x is 4, so $y = 8 \times 4^2 = 128$

Recipes

To make 4 slices of pizza	
100 ml	tomato sauce
300 g	plain flour
160 g	mozzarella cheese
2	garlic cloves
1 handfuls	basil

To work out the ingredients needed for 8 slices we would double the quantities of each ingredient.
To work out the ingredients needed for 5 slices, we first need to work out how much one slice needs.
For tomato sauce
One slice = 100 ÷ 4 = 25ml
Five slices = 25 × 5 = 125ml

Indirect Proportion

Variables are indirectly proportion if when one value doubles the other value halves.

Example if 5 men take 4 days to paint a house, then 10 men will take 2 days to paint the same house. Twice as many people, so the time halves.

We can also think of total days of work. Regardless how many men turn up to work it will take 20 working days to complete the task (5 men ×4 days).

So, if 4 men turn up to paint it will take 5 days and if only 1 man turns up it will take 20 days!

Combining ratios

If we are given two ratios such as $a : b$ and $b : c$ we can combine these to get a single ratio $a : b : c$

Example $a : b = 3 : 5$ and $b : c = 4 : 1$

To combine we need equal values of b, a bit like getting a common denominator to add fractions

Using equivalent ratios, $3 : 5 = 12 : 20$, we multiplied both sides by 4 and $4 : 1 = 20 : 5$ $(\times 5)$

Therefore $a : b : c = 12 : 20 : 5$

Quadratic sequences

$n^2 = 1, 4, 9, 16, 25 \ldots$ $2n^2 = 2, 8, 18, 32, 50 \ldots$ $3n^2 = 3, 12, 27, 48, 75 \ldots$

Quadratic sequences do not go up by the same amount, but their second difference will be the same. Second difference of $2 = n^2$, $4 = 2n^2$, $6 = 3n^2$

Example: Find the nth term of 5, 14, 27, 44, 65 ...
The first differences are 9, 13, 17, 21
The second differences are 4, 4, 4,

So it's $2n^2$ which from above is $2, 8, 18, 32, 50 \ldots$
Comparing this to the question $5, 14, 27, 44, 65 \ldots$
To get from $2n^2$ to the sequence we need to +3. +6, +9, +12, ...
This is a linear sequence (3n) so the answer is $2n^2 + 3n$

Converting currency -
if £1 is worth $1.24
When exchanging £30
you will receive
30 ×1.24 = $37.20
When exchanging $30
you will receive
30 ÷ 1.24 = £24.19

Arithmetic sequences

3, 7, 11, 15 the rule is + 4 so the next term is 19

11, 6, 1, -4 rule is – 5, the next term is -9

Geometric sequences

3, 6, 12, 24 rule is × 2, the next term is 42

81, 27, 9, 3 rule is ÷ 3, the next term is 1

Fibonacci sequence

1, 1, 2, 3, 5, 8, the next term is found by adding the two previous terms, so 5 + 8 = 13

Nth term

The nth term enables you to find any term in the sequence. N stands for the position in the sequence. So for the first term n=1, for the second term n=2, third term n= 3 and so on.

If the nth term is 5n – 2, the first term is 5 × 1 – 2 = 3, the second term is 5 × 2 – 2 = 8 and the third term is 5 × 3 – 2 = 13. Therefore the sequence 5n – 2 is 3, 8, 13, 18, 23 ... notice the rule is + 5

Sharing in a ratio

Share £10 between Steve and Jon in the ratio 3 : 2



There are 5 parts altogether. £10 ÷ 5 = £2.
So each part is worth £2.



Therefore Steve gets 3 × £2 = £6
Jon gets 2 × £2 = £4

Finding the nth term

The sequence 7, 12, 17, 22 goes up by 5 each time, therefore the nth term is 5n + something
5n is 5, 10, 15, 20, 25,
therefore 7, 12, 17, 22 is 5n + 2.

The sequence 4, 10, 16, 22 goes up by 6,
6n is 6, 12, 18, 24, 30
therefore 4, 10, 16, 22 is 6n – 2

Sequences to recognise

1, 4, 9, 16, 25, 36 ... square numbers

1, 8, 27, 64, 125 ... cube numbers

1, 3, 6, 10, 15, 21 ... triangle numbers

2, 4, 8, 16, 32, 64 ... powers of 2

3, 9, 27, 81, 243 ... powers of 3

Simplify Ratios by dividing each number by their highest common factor.

15 : 35 (÷5) gives 3 : 7

30 : 24 (÷ 6) gives 5 : 4

Best buy

OFFICE DEALS	PAPER WORLD
Packs of 20 folders £10.80	Pack of 15 folders £8.40

To decide which the best value is, we need to compare the same quantity.

If we bought 60 folders, it would cost:
3×£10.80 = £32.40 at Office Deals
4 ×£8.40 = £33.60 at Paper World.
Therefore Office Deals is cheaper.

Alternatively we could find how much 1 folder costs at each store.
At Office Deals £10.80 ÷ 20 = £0.54
At Paper World £8.40 ÷ 15 = £0.56
Again Office Deals is cheaper.

Solving problems with ratios

When solving ratio problems it is often easiest to draw boxes to represent parts. For example: Sabrina and Vicky share money in the ratio 2 : 5. Vicky gets an extra £15.



If the 3 parts are worth £15 then each part is £15 ÷ 3 = £5

Therefore filling in the boxes shows us that
Vicky gets 2 × £5 = £10 and Sabrina gets 5 × £5 = £25

Unitary Method

When prices are in proportion we can find the cost of buying a different quantity by first finding the price of one item.

If 4 pens cost £1.20, what do 7 pens cost?

4 pens = £1.20

1 pen = £1.20 ÷ 4 = 30p

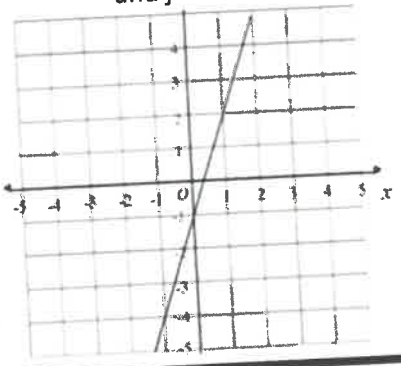
7 pens = 7 × 30p = £2.10

Plotting graphs

To plot y = 3x – 1 use a table to find the coordinates

x	-1	0	1	2
y = 3x – 1				

When x = 2, y = 3 × 2 – 1 = 5,
Therefore one of the coordinates is (2,5)
These coordinates can be plotted
and joined with a straight line



Equation of a straight line

The general form of the equation of a straight line is

$$y = mx + c$$

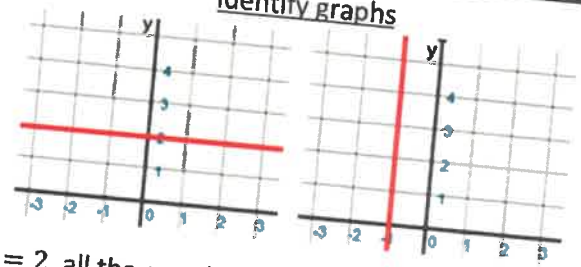
The gradient, m tells us how steep the line is.

$$m = \frac{\text{Change in Y}}{\text{Change in X}}$$

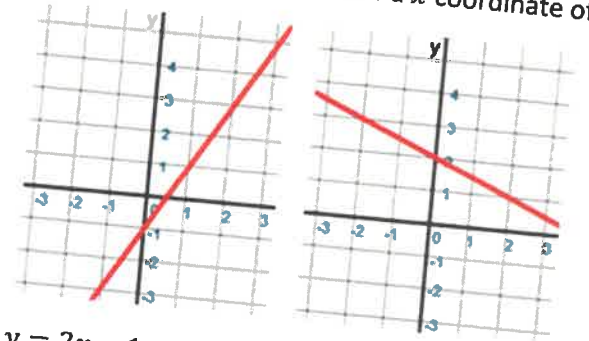


The *y intercept, c is* where it crosses the y axis.

Identify graphs



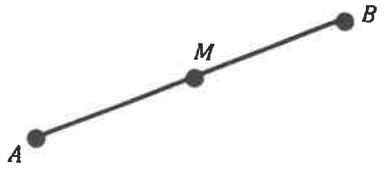
$y = 2$, all the coordinates have a y coordinate of 2.
$x = -1$, all the coordinates have a x coordinate of –1



$y = 2x - 1$, gradient is 2 and y-intercept is –1
$y = -\frac{1}{2}x + 2$, gradient is $-\frac{1}{2}$ and y-intercept is 2

Midpoint of a line

The middle of a line is its midpoint
The formula is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

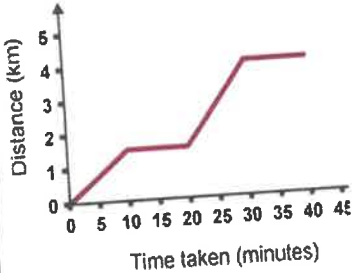


If $A = (2,5)$ and $B = (8,6)$

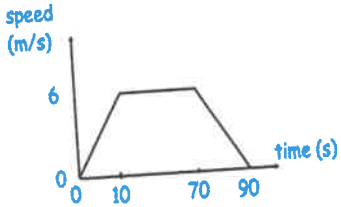
Then the midpoint is half the sum of the x coordinate and half the sum $\left(\frac{2+8}{2}, \frac{5+6}{2}\right) = (5, 5.5)$

Real-life graphs

We can plot the values of any 2 variables. Common examples include: distance-time graphs or velocity-time graphs



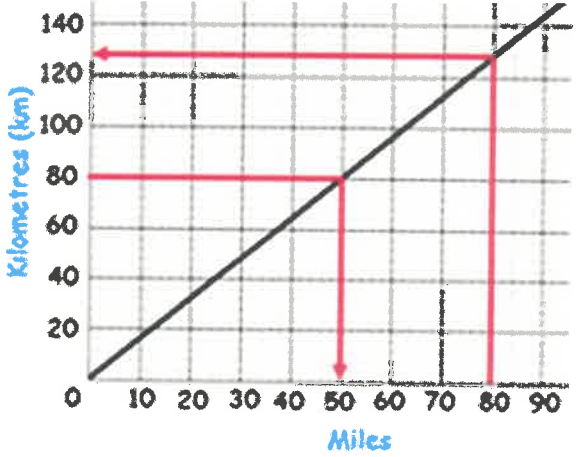
The gradient is the speed the object is travelling at. The steeper the graph, the faster the object. Horizontal lines have a speed of 0, the object is stationary.



The gradient is the acceleration of the object. A negative gradient means its decelerating (slowing down). Horizontal lines have an acceleration of 0, the object is travelling at a constant speed. The area under the graph is the distance travelled.

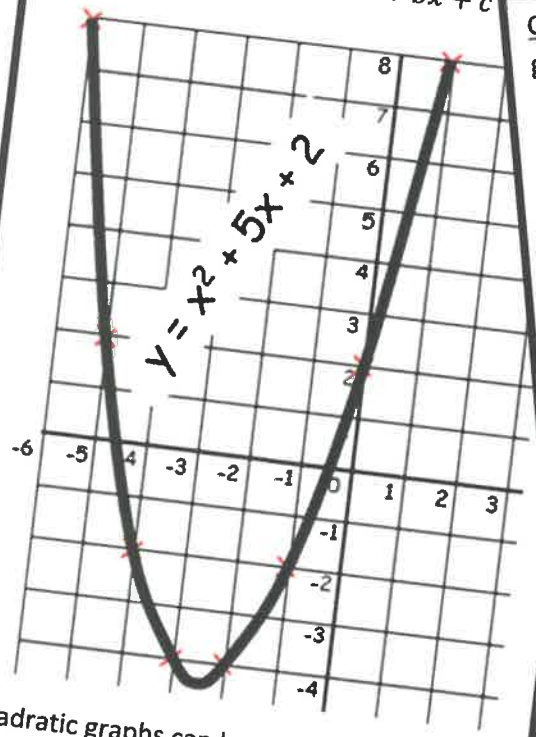
Conversion graphs

This graph converts kilometres to miles



We can use the graph to show that 80km is 50 miles or that 80 miles is 128km

Quadratic graphs - $y = ax^2 + bx + c$

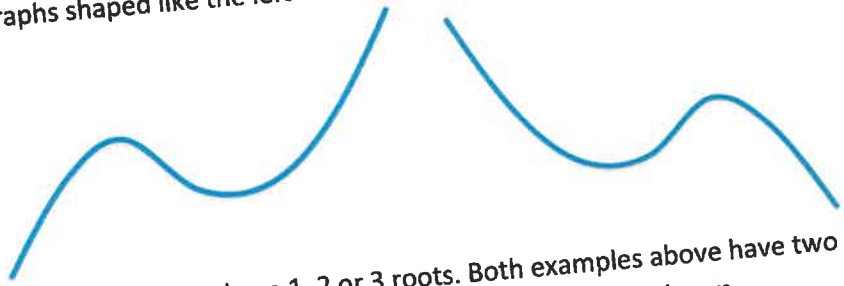


Quadratic graphs can have 0, 1 or 2 roots (the points where it crosses the x axis). Roots can be found by factorising (higher only).

If the x^2 coefficient is positive the graph is U shaped (negative is ∩ shaped).

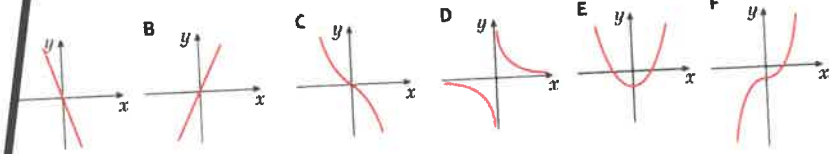
The turning point is where the graph turns, in a U shaped graph it is known as a minimum (it is the lowest value) and in a ∩ shaped graph it is known as a maximum. Turning points can be found by completing the square (higher only).

Cubic graphs - $a + bx^2 + cx + d$ - positive coefficients of x^3 give graphs shaped like the left example, negative are like the right example



Cubic graphs can have 1, 2 or 3 roots. Both examples above have two turning points, 1 is a minimum and 1 is a maximum

Recognise graphs by their shape



Matching the graphs above to each equation

$y = x^2 - 1$. This is a quadratic graph so is U shaped. The –1 means it intercepts the y axis at –1. Therefore it must be graph E.

$y = x^3 - 2$. This is a cubic graph. The –2 means it intercepts the y axis at –2. Therefore it must be graph F.

$y = 2x$. This is a linear graph with a positive gradient – graph B

$y = \frac{1}{x}$. This is a reciprocal graph – graph D

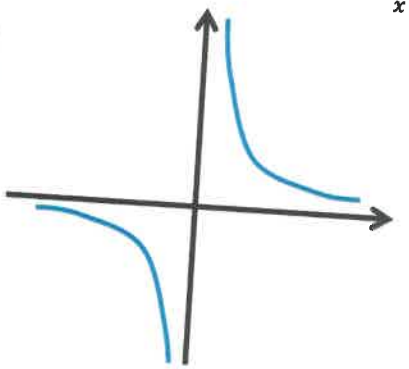
$y = -x^3$. This is a cubic graph, being negative it will look like graph C

$y = -3x$. This is a linear graph with a negative gradient – graph A

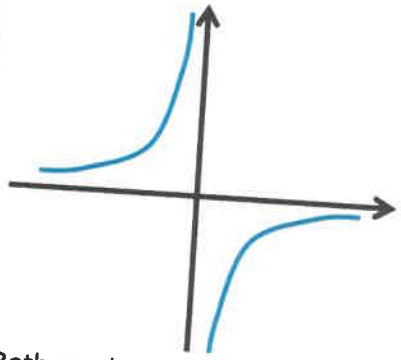
There are also circle, exponential and trig functions to cover in Yr 11

Reciprocal graphs $y = \frac{a}{x}$

If a is positive, for example $y = \frac{1}{x}$



If a is negative, for example $y = \frac{-2}{x}$



Both graphs have no values on either axes, the lines get infinitely close but never touch. This is because they have asymptotes.

When $x = 0$, y does not exist because dividing by 0 is impossible.